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### Abstract

Dispersion of the phase-coefficient and the wave-impedance of slot-line, coplanar-strip-line, suspended-substrate line, and six different fin lines is calculated in one uniform treatment applying Galerkin's method on the electric interface fields. Results are presented at fin-line examples in the 8 and 3 mm wavelength regions, showing advantageous properties of the considered structures, especially of the newly presented antipodal-fin line, which allows an impedance variation from about  $10 \Omega$  to over  $400 \Omega$ . Tested applications are detectors, PIN-attenuators, coax to dielectric-line transition, and balanced-signal mixer, all for broadband application.

### Introduction

Different quasi-planar and planar structures have been proposed for millimeter-wave applications. The slot-line and the coplanar-strip line have been treated sufficiently<sup>1,2</sup>. Fin-lines have long been applied<sup>3-5</sup>, but no theoretical approach has been available until recently<sup>6,7</sup>. For the suspended-substrate line only quasi-static results have been published<sup>8,9</sup>. It is, however, possible to calculate all these structures in one uniform treatment applying Galerkin's method<sup>7</sup>.

Fig. 1 depicts the considered geometries: a, b, c show the unilateral, bilateral, and antipodal fin-lines respectively, where the fins are contacted to the waveguide walls. The corresponding versions with isolated fins are to be seen in Fig. 1 d, e, f. Slot-, coplanar-, strip-, and suspended-substrate-lines are shown in Fig. 1 g, h, i. Fin- and suspended-substrate lines are considered quasi-planar, since components are modelled by strip structures leaving the hollow waveguide unchanged. It is sufficient to deal with the fin-lines in further, as the other forms can be deduced from those. It is supposed that the fins are infinitely thin and all media loss-free.

### Fourier Representation of the Fields

In applying Galerkin's method the field-spaces of the three fin-line types are divided into subregions as Fig. 2 shows, where electric and magnetic walls account for the symmetries of the fundamental mode. In each of the subregions the Helmholtz-equation is solved, including the satisfying of any boundary condition. A complete solution can be achieved via the  $E_x$  and  $H_x$  components only. For brevity only the terms for the antipodal fin-line shall be given as an example, the fields of the other fin-lines being very similar:

$$E_x^{(1)} = \sum_n A_n^{(1)} \cos \bar{\alpha}_n (x-a) \sin \alpha_n y,$$

$$E_x^{(2)} = \sum_n A_n^{(2)} \left\{ \delta_e \cos \tilde{\alpha}_n x \sin \alpha_n y + \delta_o \sin \tilde{\alpha}_n x \sin \alpha_n y \right\}, \quad (1)$$

in  $x \geq 0$

with

$$\delta_e = \begin{cases} 0, & n \text{ even} \\ 1, & n \text{ odd} \end{cases}, \quad \delta_o = \begin{cases} 0, & n \text{ odd} \\ 1, & n \text{ even} \end{cases}, \quad (2)$$

and the transversal phase-coefficients

$$\alpha_n = n \pi / 2b,$$

$$\bar{\alpha}_n = \sqrt{k_0^2 - \beta^2 - \alpha_n^2}, \quad (3)$$

$$\tilde{\alpha}_n = \sqrt{k_0^2 \epsilon_r - \beta^2 - \alpha_n^2},$$

where  $k_0$  and  $\beta$  are the phase-coefficients of free space and of the line respectively. In eqn. (1) only the subregions  $x \geq 0$  are needed because of the symmetry condition

$$E_x(-x, 2b-y) = E_x(x, y), \quad H_x(-x, 2b-y) = H_x(x, y). \quad (4)$$

The  $H_x$ -components are obtained by replacing sine by cosine, cosine by sine and naming the Fourier-coefficients  $B_n^{(i)}$ .

### Continuity- and Boundary Conditions

Applying all continuity conditions at the interface gives all Fourier-coefficients in terms of the fin current  $I$ :

$$A_n^{(i)}, B_n^{(i)} = f \left\{ L_{1n}(I_z), L_{2n}(I_y) \right\}, \quad (5)$$

where the linear operator  $L$  is given by

$$L_{1n}(\xi(t)) = \int \xi(t) \sin \alpha_n t \, dt, \quad (6)$$

$$L_{2n}(\xi(t)) = \int \xi(t) \cos \alpha_n t \, dt,$$

the integral taken over the considered intervall.

It can be shown that in terms of convergence it is advantageous to apply Galerkin's method on the slot-fields rather than on the fin-currents<sup>7</sup>. So consider the remaining boundary conditions in the formulation

$$\begin{aligned} E_y^{(n)} &= f(y), \\ E_z^{(n)} &= g(y), \end{aligned} \quad \text{at } x=d, \quad 0 \leq y \leq \left\{ \frac{b}{2b} \right\}, \quad (7)$$

where  $f, g$  represent the fields in the slot and are zero elsewhere. Eqn. (7) gives the Fourier-coefficients of region 1 as functions of  $f$  and  $g$ :

$$A_n^{(i)}, B_n^{(i)} = \left\{ L_{2n}(f), L_{1n}(g) \right\}. \quad (8)$$

On eliminating  $A_n^{(i)}, B_n^{(i)}$  eqns. (5) and (8) can be rearranged to read

$$\begin{aligned} \Lambda_{11} L_{2n}(f) + \Lambda_{12} L_{1n}(g) &= L_{2n}(I_y), \\ \Lambda_{21} L_{2n}(f) + \Lambda_{22} L_{1n}(g) &= L_{1n}(I_z), \end{aligned} \quad (9)$$

where  $\Lambda_{ij}$  are well-defined functions. Summing eqn. (9) gives

$$\begin{aligned} \sum_n \left( \frac{1}{\epsilon_n} \Lambda_{11} L_{2n}(f) \cos \alpha_n y \right. \\ \left. + \Lambda_{12} L_{1n}(g) \cos \alpha_n y \right) &= \frac{1}{2} \left\{ \frac{b}{2b} \right\} I_y(y), \\ \sum_n \left( \Lambda_{21} L_{2n}(f) \sin \alpha_n y \right. \\ \left. + \Lambda_{22} L_{1n}(g) \sin \alpha_n y \right) &= \frac{1}{2} \left\{ \frac{b}{2b} \right\} I_z(y), \end{aligned} \quad (10)$$

with

$$\delta_n = \begin{cases} 2, & n=0 \\ 1, & n \neq 0. \end{cases} \quad (11)$$

Galerkin's method is applied to eqn. (10) by writing

$$\begin{aligned} f(y) &= \begin{cases} \sum_m C_m \cos \eta_m (y-w), & w \leq y \leq \left\{ \frac{b}{2b} \right\} \\ 0, & 0 \leq y \leq w, \end{cases} \\ g(y) &= \begin{cases} \sum_m D_m \sin \eta_m (y-w), & w \leq y \leq \left\{ \frac{b}{2b} \right\} \\ 0, & 0 \leq y \leq w, \end{cases} \end{aligned} \quad (12)$$

$$\eta_m = m \pi / s$$

and testing with  $\cos \eta_m (y-w)$  and  $\sin \eta_m (y-w)$ . The right-hand sides of eqn. (10) will vanish when testing, since  $I_y, I_z$  and  $f, g$  are defined in complementary field spaces. Thus the fin currents are eliminated from the system and the problem is formulated in the slot fields only.

The vanishing of the determinant of the resulting doubly-infinite system of homogeneous equations is condition to determine the phase-coefficient  $\beta$ . Then in turn any desired field-components can be calculated. Further details concerning relative convergence, edge-condition, convergence rapidity, and the solutions for the isolated-fin case can be found in<sup>7</sup>.

## Results

Results for  $\epsilon_{\text{eff}}$  and  $Z_L = U / I$  are presented in Figs. 3 and 4 for two representative substrates and frequency ranges. Dispersion of the fin-lines is very small and can in the case of the wave impedance often be neglected. The small dispersion and the lower cut-off frequency compared to the corresponding hollow waveguide propose the fin-lines for broadband applications. Of interest is the range of realisable wave impedances. Uni- and bilateral-fin lines have a  $Z_L$  low limit of about  $100 \Omega$  because of the minimum slot-width achievable by etching. The antipodal fin-line, however, changes from slot-line behaviour ( $s > 0$ ) to parallel-plate-line behaviour ( $s < 0$ , overlapping). So a range of about  $400 \Omega$  to below  $10 \Omega$  of wave-impedance can be realised. Three applications employing advantageous combinations of antipodal fin, unilateral fin, suspended substrate, and microstrip lines are shown in Fig. 5 as they have been tested in laboratory: broadband detector with quarterwave-transformers, broadband coax to D-line transition via microstrip and antipodal-fin line, and a broadband balanced-signal mixer.

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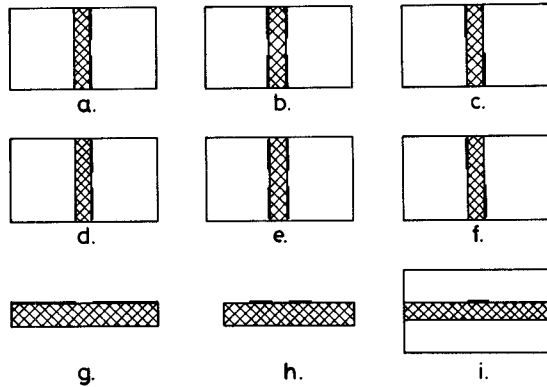


Fig. 1

Fin-line geometries and related structures to be treated in one uniform formulation: (a) unilateral fin, (b) bilateral fin, (c) antipodal fin; (d), (e), (f) isolated counterparts of (a), (b), and (c); (g) slot, (h) coplanar-strips, (i) suspended substrate.

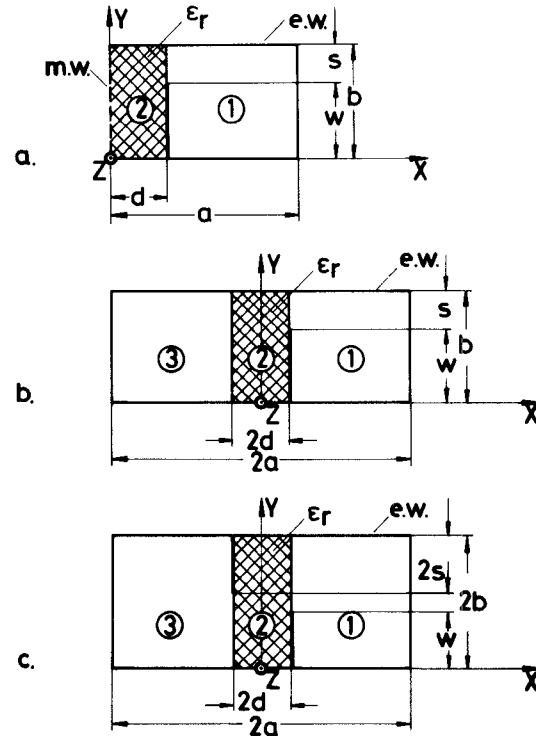


Fig. 2

Division of the field space into sub-regions; e.w.: electric wall, m.w.: magnetic wall; (a) bilateral fin, (b) unilateral fin, (c) antipodal fin.

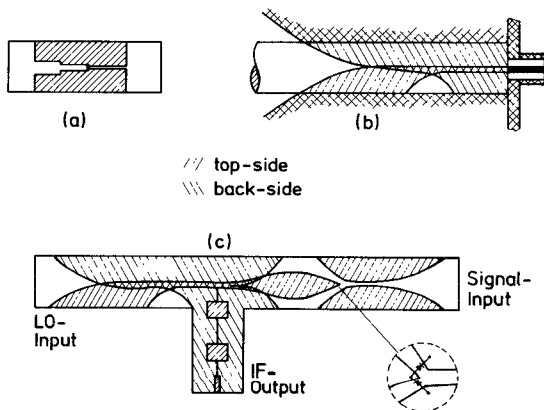


Fig. 5

(a) Broadband fin-line detector, (b) broadband coax to D-line transition, (c) broadband balanced-signal mixer.

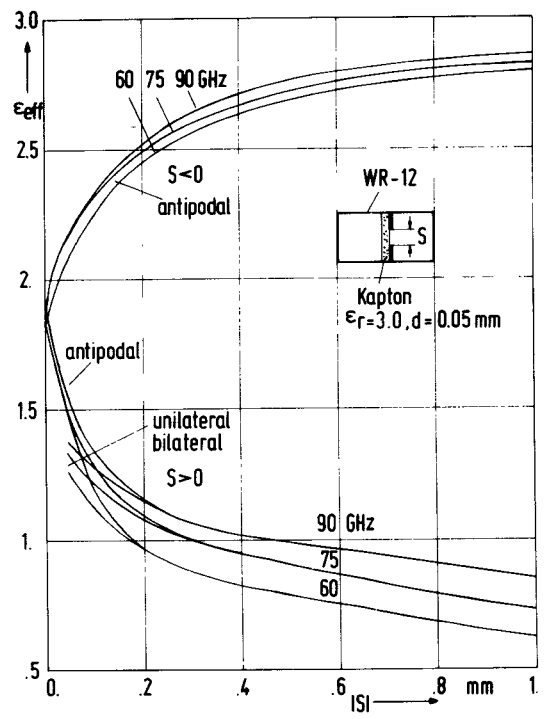
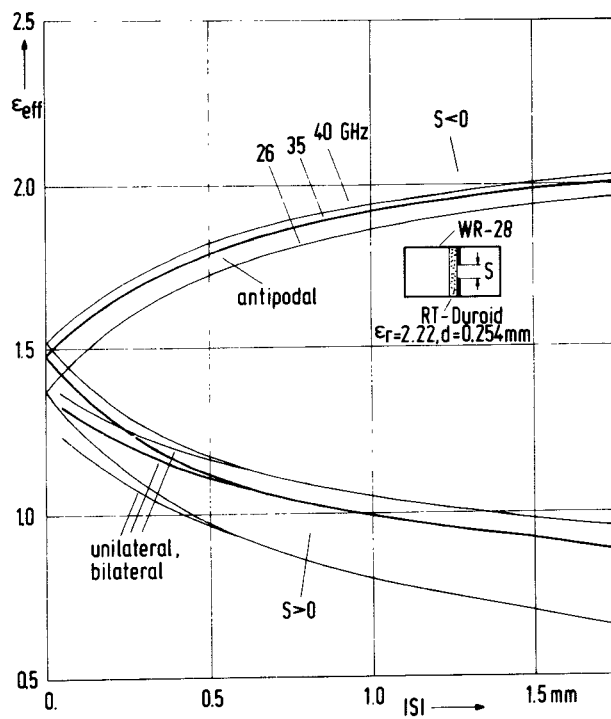


Fig. 3

Effective dielectric constant  $\epsilon_{\text{eff}}$  vs.  
slot-width  $s$ . WR-28: 7.112 mm \* 3.556 mm,  
WR-12: 3.099 mm \* 1.549 mm

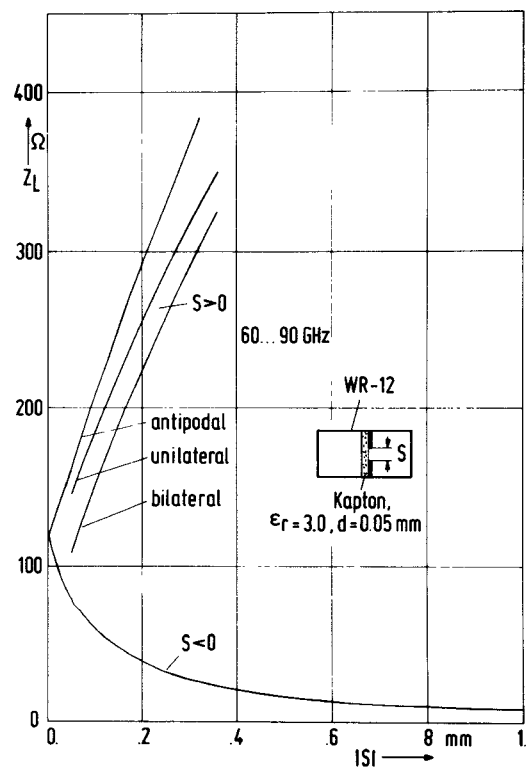
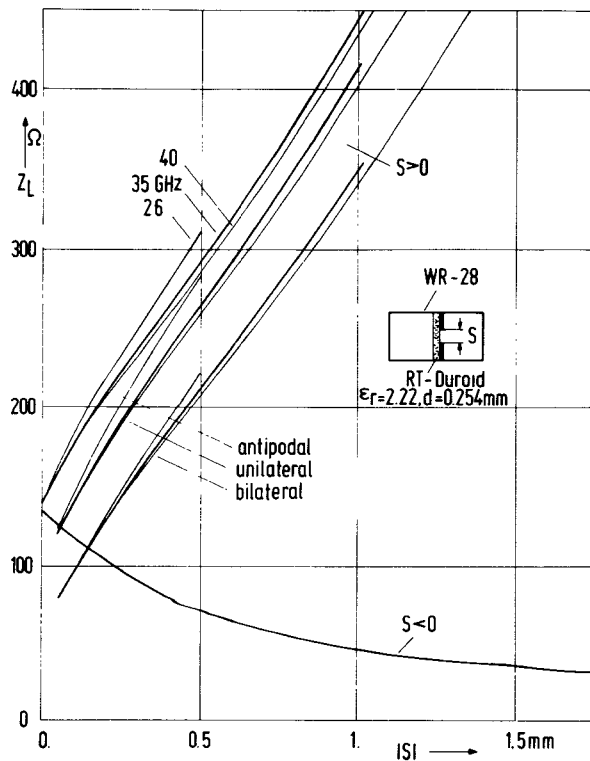


Fig. 4

Wave-impedance  $Z_L$  vs. slot-width  $s$ .  
WR-28, WR-12 see Fig. 3